Session 2559

A Lecture on Accurate Inductive Voltage Dividers

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Introduction

The United States Naval Academy is an undergraduate school with a successful engineering program. An underlying philosophy in the Systems Engineering Department is to emphasize hands-on experience while maintaining a solid theoretical background. To realize this principle, both teachers and students face many challenges. In this paper an attempt is made to teach students how to build a very accurate ac voltage divider with an uncertainty better than a part-per-million. This implies building a transformer-based divider. The idea is to bridge the gap between the state-of-the-art achievements in modern research and the undergraduate level of expertise. Several years ago a group of midshipmen built a voltage divider for a critical point experiment that was conducted in collaboration with a National Institute of Standards and Technology and NASA space shuttle program. In order for the students to be able to build a good transformer lots of coaching took place. This lecture is intended to introduce an engineering student into the art of precise voltage-ratio measurements.

There are a number of applications that use very precise voltage-ratio devices, but one of the most common is in impedance bridges. Measurement bridges are constructed in such a way that they allow comparison of a standard unit and a unit under test. The standard and the device under test are connected in series and the impedance of a device under test is calibrated by finding the voltage ratio of the serial connection (see Figure 1.). Impedance of resistors, capacitors, and inductors is tested using alternating current. Very often experiments in physics use the same concept to determine the properties of materials. The capacitance seems to be the most often measured quantity. In this paper we will concentrate on presentation of voltage-ratio devices. From Figure 1 it can be seen that the voltage ratio is obtained using an inductive voltage divider.

There are a number of references that give technical details on inductive voltage divider design [1] [2]. But they are written by engineers deeply involved in research, for engineers doing the same. Our goal is to present a straightforward and comprehensible lecture on voltage divider design.

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Ideal Transformer

The transformer's principal of operation is simple. An alternating current through a conductor creates a magnetic field around the conductor. We can use Ampere's Law to find the magnetic induction field, B, around a conductor in a vacuum:

$$\mathbf{B(r)} = \mu_0 \mathbf{I} / (2\pi \mathbf{r}) \qquad \text{[units: Tesla]} \qquad \qquad \dots(1)$$

where
μ_0 is the magnetic permeability of the vacuum, $4\pi \times 10^{-7}$ [H/m] or [Wb/Am]
r is a radius where the field is calculated [m],
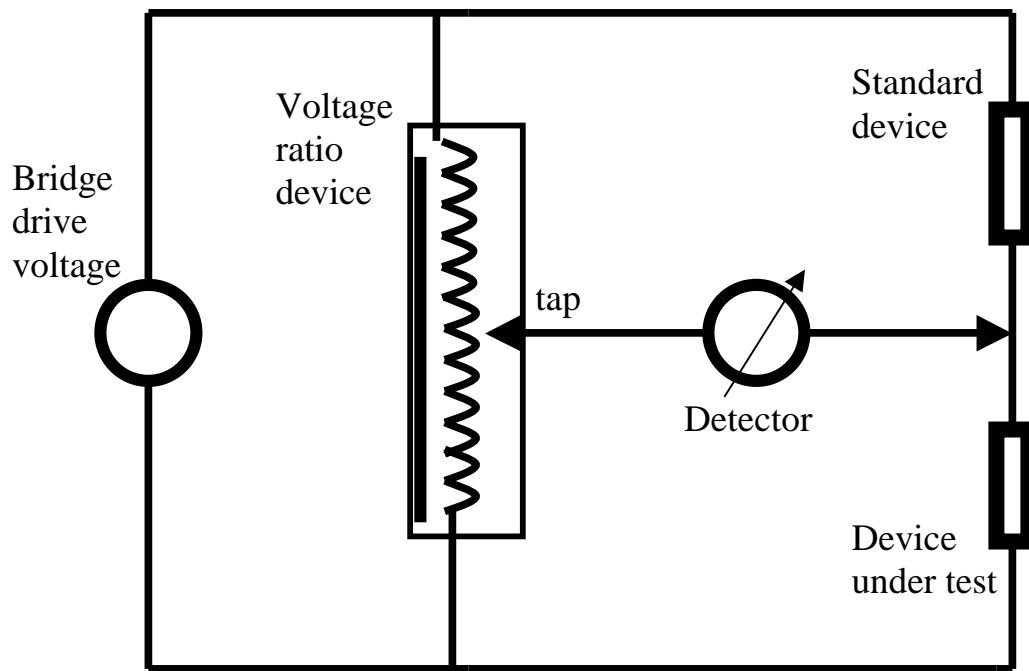I is the current applied [A].

Figure 1. Simplified Measurement Bridge.

If we put a conducting loop in this field, an electromotive force will be induced in it. Faraday's Law of electromagnetic induction states that an electromotive force (voltage) is induced in a conductor due to the change of the electromagnetic flux (Φ) with time.

$$\mathbf{emf} = - \delta\Phi(\mathbf{t})/\delta\mathbf{t} \ \ [V] \qquad \dots(2).$$

Electromagnetic flux through a given area is defined as an integral over the area of the scalar product of magnetic field vector and the area element vector. From Ampere's law we see that the

intensity of the magnetic field created is proportional to the current applied. The magnetic field intensity is in a sense a measure of the "effort" that the current is putting into the establishment of the magnetic filed. The voltage induced is proportional to the flux enclosed by the loop. In this fashion we have *transformed* the energy created by a conductor to a loop, using the alternating electromagnetic field.

This is a general principle and in practice several steps are introduced to better control and, consequently, to better predict this transformation process. In practice, transformers are wound on high permeability cores constructed of steel alloys in order to increase the field intensity. Inductive voltage dividers used for high accuracy measurements are wound on toroidal cores because it is relatively easy to calculate the field generated within the core (given for the rectangular cross-section):

$$\mathbf{B(r)} = \mu_0\mu_r\mathbf{NI} / (2\pi\mathbf{r}) \quad [\text{Tesla}] \dots(3)$$

where μ_r is the relative permeability of the material used to make core,
N is the number of turns.

From this equation it is easy to calculate the flux through a cross-section of a core:

$$\Phi(\mathbf{t})= \mu_0\mu_r\ \mathbf{NI(t)\ h\ ln(b/a)} / (2\pi) \qquad [\text{Weber}] \qquad\qquad \dots(4)$$

where N is the number of turns,
b is the outside core diameter [m],
a is the inside core diameter [m],
h is the core height [m]
ln is natural logarithm.

All of the presented equations point to the fact that the induced voltage in a transformer is proportional to the number of turns. In the ideal case when all of the windings enclose all of the flux, the voltage ratio, R=Vin/Vout, is defined as:

$$\mathbf{R=(Number\ of\ turns\ at\ the\ tap)/(Total\ number\ of\ turns)} \qquad \dots(5)$$

The tap is an in-between point that determines the voltage ratio (see Figure 1). There are a number of factors that influence the conditions under which the electromotive force is induced. It is essential to understand that uniformity in distribution of electrical and magnetic parameters among the windings contributes to the ratio errors. The deviations from this exact value constitute ratio errors.

Let us consider some analogies between electrical and magnetic circuits. In the electrical circuits electromotive force (voltage) drives the current flow. Magnetomotive force of the magnetic circuit is equal to the effective current flow applied to the core. It is measured in ampere-turns. In the electrical circuits conductors that carry current are separated from one another with some kind of insulating material. A moderately good insulator such as rubber has conductivity 10^{-20} times the conductivity of copper. This means that in general conduction current in the insulating

material is negligibly small in comparison with the current in the conductor. In the magnetic world the most diamagnetic substance (magnetic insulator) is bismuth and it has permeability 0.9998 times the permeability of air. Practically air is a flux insulator and that means that most of the time ferromagnetic material has only several hundreds times better permeability. The flux that is not confined to the core is called leakage flux.

At this point the properties of an ideal transformer are given in order to provide the design guidelines. We will try to match these ideal conditions in practice. An ideal transformer has negligibly small resistance and capacitance of the windings and negligibly small core loss. Also, in the ideal transformer, the entire magnetic flux links all the turns of both windings and the permeability of the core is so high that a negligibly small magnetomotive force produces the required flux.

Real transformer

Let us consider the conductor used to wind the transformer. In the ideal case, if its resistance is zero, there is no voltage drop along the wire. However, in the practical case, the actual resistivity of the wire and its cross-sectional area determines the resistance. The thicker the wire, the lower the resistance! In practice, small imperfections in wire dimensions and resistance could make a difference in a divider's voltage ratio at the part-per-million level.

In order to minimize the effect of the conductor's manufacturing flaws, inductive voltage dividers are wound using a rope. Usually, if a decade divider is made 10 wires are twisted to make a rope that is wound around the core. The individual wires of the rope are connected in series and the output voltage is tapped. The goal is to have uniform sections. Using thick wire tightly twisted in a rope to wind high precision dividers is a technique that addresses negligibly small resistance in the ideal transformer.

It is beneficial to have a rope of twisted strands in order to achieve even distribution of winding resistance, but we also have to analyze the capacitance in such a case. It is obvious that the proximity of adjacent turns increases the capacitance. The wire used to wind transformers is coated with thin insulation. The voltage between two adjacent turns is proportional to the applied voltage divided by the number of turns. This voltage could be significant. All of these conditions suggest that at higher frequencies the capacitive currents are the predominant factor in ratio errors. The easiest way to reduce the capacitance between the adjacent turns is to have them widely spaced. This is not always possible due to core size limitations. The best-case scenario is to have negligible inter-winding capacitance. In the case of the inductive voltage divider it is acceptable to achieve minimal possible capacitance providing that all of the sections have the same capacitance value. The technique that achieves this condition is presented in the section titled Reducing Admittance Load of the Inter-Winding Capacitances.

Let us consider a core used to build voltage dividers. It influences several ideal transformer properties. In practice, supermalloy cores are used. This material is also known as 80% nickel-iron alloy (79% nickel, 17% iron, 4% molybdenum). This alloy is used in tape wound cores and has low core and high initial relative permeability. In our discussion about electromagnetic fields, it was stated that the electromotive force is induced in a conductor placed in the field. The

core is made of a conductive material and currents (eddy) are induced in the core. This is the cause of core loss. In order to minimize the loss, cores are built from thin metal sheets isolated from each other and wound like a tape role. Usually 1-mil or 2-mil tape thickness is used. The drawback of this approach is that, effectively, the amount of core material enclosed by the windings is reduced and, consequently, more windings are necessary for the same operating flux.

In practice, the electromagnetic field in a transformer is changing with sinusoidal periodicity. Ferromagnetic cores are magnetized in this fashion and a certain amount of energy is required to establish the necessary flux. More energy is used on core magnetization when the field is changing faster. The energy necessary for core magnetization is proportional to the area under the hysteretic curve and the operating frequency. As in the core loss problem, the bottom line in our discussion is the current flow. In order to minimize the magnetomotive force needed to produce the necessary flux, the core material must have high permeability. Again, supermalloy is a good choice. Supermalloy has an initial permeability of several tens of thousands and it gradually decreases at higher frequencies. These values are not enough to achieve part-per-million accurate voltage ratios. To further decrease the ratio errors, a technique was developed by Cutkosky [2] to build transformers in two stages. The next section will address the performance of a two-stage transformer.

Two-stage transformers

(a)   (b)   (c)
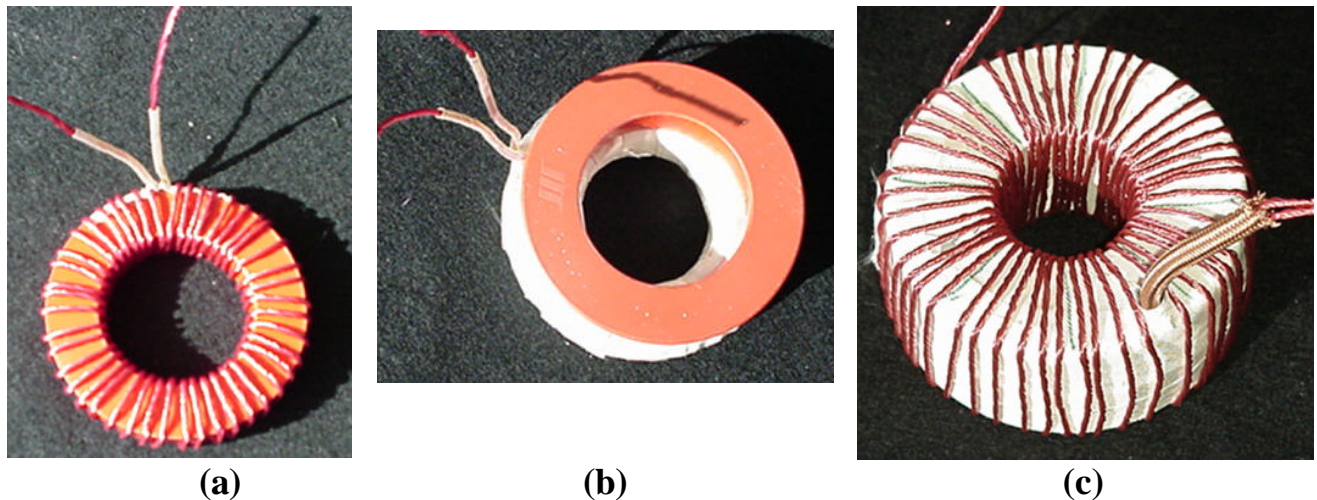
Figure 2. Design of a two-stage transformer. (a) Magnetizing winding. (b) Magnetizing winding and the second core. (c) Ratio winding.

We have mentioned the need to minimize the current flow necessary for producing certain electromagnetic field characteristics. Let us consider the situation where one core is wound with a layer of windings. A second core is placed on top of the first core and the same number of turns is wound around both cores. (See figure 2). So what is achieved with this arrangement? We have the first stage (magnetizing) winding and the second stage (ratio) winding. When the magnetizing winding is energized a magnetic field is created in the surrounding space. The

current in the magnetizing winding fully magnetizes the first core and for the most part it magnetizes the second core. Due to the fringing field loss, the second core will not have the exact same flux. In this way the burden of magnetization of the first core and most of the magnetization of the second core is transferred from the ratio winding to the magnetizing winding. This is important because only the current flow in the ratio winding determines the voltage ratio and in the two-stage transformer this current flow is minimized. Now the ratio winding has only to supply the rest of the flux needed for full operating voltage. In this way, most of the core loss is also transferred to the magnetizing winding. Both magnetizing and ratio windings are energized from the same source but we use two sets of leads. Most of the current flows through the magnetizing leads. The voltage ratio is derived from the ratio winding and errors of the two-stage inductive voltage divider are reduced significantly.

There is one other practical aspect to consider when building a good inductive voltage divider. That is the magnetic and electrostatic shielding. Usually, a two-stage transformer is enclosed within a mu-metal toroidal box. (See Figure 3.) Mu-metal is a ferromagnetic material with very high permeability. Ratio turns are wound around the toroidal box with cores in it. This magnetic shield is used to further confine the magnetic field generated by the magnetizing winding, so that the ratio windings enclose almost all of the flux generated. Electrically, this box is connected to the magnetizing winding potential. Special care is introduced to prevent the mu-metal box from creating a shorted turn since mu-metal is a conductive material.
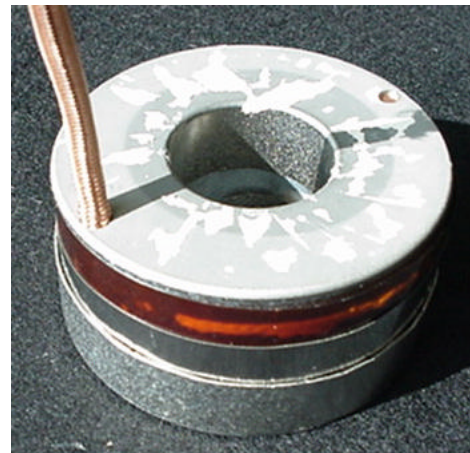
(a)   **(b)**

Figure 3. Magnetic shielding. (a) Mu-metal toroidal boxes. (b) Mu-metal shielding isolation.

Reducing the Admittance Load of the Inter-Winding Capacitances

The admittance load of the inter-winding capacitances between the different parts of the winding degrades the ratio accuracy of an inductive voltage divider. A method to optimize the divider's

internal admittance load was suggested by Lu of the Chinese National Metrology Institute and Klonz and Bergeest of the German National Metrology Institute [3].

A practical way to equally distribute the capacitances is to twist insulated copper wires to form a rope ~~of strands~~. Let us assume that all ***distributed*** capacitances between all of the strands of such a cable are equal. Then it could be shown that total equivalent ***concentrated*** capacitances at the divider's taps would be different. The highest values of capacitances will be in the middle section. Those concentrated capacitances are calculated from all the distributed capacitances between all the strands to each other.

Let us assume that we have a divider that is wound using a rope of 3 strands (wires 1, 2 and 3). When a wire is wound around a core it is important to label the in and out. This labeling is necessary for the tap formation using out of one wire (strand) and in of a different wire (strand). Using a capacitance meter the distributed capacitances could be measured between each strand: C12, C13 and C23 (see Figure 4.a). In figure 4.b, three strands of wire are connected in such a way that four taps are made (x, y, z and w). It has been shown that a distributed capacitance between two strands, Cdist, can be replaced by a concentrated capacitance, Cconc=Cdist/2, which appears between respective strands' ins, and a concentrated capacitance Cconc=Cdist/2, that appears between respective strands' outs. This step is shown in Figure 4.b. Figure 4.b shows concentrated capacitances between the strands' ends and since the taps are labeled, the arrows point to the higher potential. It is easy to combine capacitances C12/2 and C23/2 since those are connected in parallel. But in order to simplify the scheme we will have to use an approximation for the capacitance C13. The simplest assumption would be to have the same capacitance between each tap C13 spans. Figure 4.c shows the application of this assumption. It is important to note that Figure 4.c shows concentrated capacitances between the taps. Tap w is actually ground.
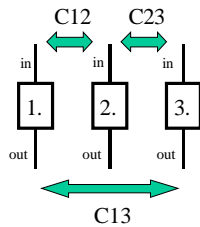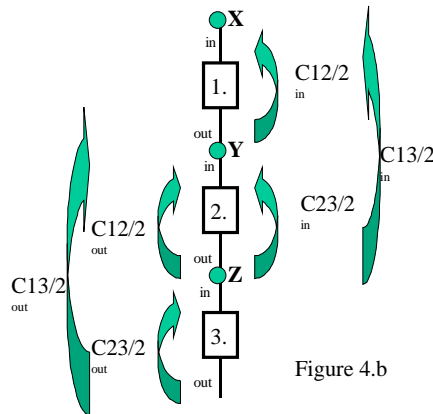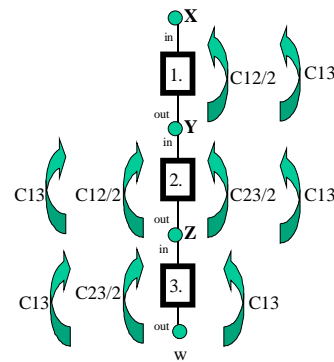

Figure 4. Capacitance transformation. (a) Distributed capacitance measurements using a capacitance meter. (b) Concentrated capacitances between wires. For example C13/2 is concentrated capacitance between strand 1-in and strand 3-in. (c) Equivalent concentrated capacitances between the taps.

We are now ready to write the equations for the concentrated capacitance at each tap (x,y,z) based on the measured distributed capacitances:

$$Cx = (C12 + 2C13)/2$$
$$Cy = (C12 + C23 + 4C13)/2$$
$$Cz = (C23 + 2C13)/2 \qquad ....(6)$$

In this very simple example it is easy to see that since there are 3 wires, they can be arranged in 6 different ways to form taps. Figure 2.c shows the order 1 2 3. It is possible to perform calculations for each permutation and arrive at 6 different sets of tap concentrated capacitances. Our goal is to minimize the divider ratio errors at each tap. That is achieved if tap capacitances are as small as possible and as equal to each other as possible. From our 6 calculations we should choose the set where maximum tap capacitance is minimal. That choice fixes the wire arrangement for the taps.

Let us work out this procedure using an example. Measured distributed capacitances: C12=123_pF, C13=987_pF and C23=67_pF.

Meas. Order	Wire Order	Capacitance between the taps X and Y [pF]	Capacitance between the taps Y and Z [pF]	Capacitance between the taps Z and W [pF]	Maximum Capacitance [pF]
1.	3 2 1	245	435	190	435
2.	2 3 1	235	420	185	420
3.	3 1 2	295	585	290	585
4.	1 3 2	185	420	235	420
5.	2 1 3	290	585	295	585
6.	1 2 3	190	435	245	435

Table 1. Concentrated capacitences for each wire order.

Let us find the maximum capacitance for each wire order. It could be seen from the table that values in the middle column range from 585 pF to 420 pF. The permutations are symmetrical so we can pick order 2 3 1. This particular order means that the new tap X is created from the second wire in. The new tap Y is created from the second wire out and the third wire in. The new tap Z is created from the third wire out and the first wire in. The tap W (ground) is created from the first wire out.

The next step is to add external capacitors to the other taps in order to achieve the same capacitance at each tap. So an external capacitor is put between the taps X and Y of 185 pF (420-235). Also a capacitor of 235 pF is placed between the taps Z and W. In this way all of the taps have the same capacitance 420 pF and it is the smallest possible for the strands at hand.

It is now possible to extend this process to any number of wires in a strand. It is not a trivial procedure but it is straightforward and could be very well organized using matrices. The exhaustive set of measurements of mutual capacitance of all wires that form taps is organized in

a matrix. Equivalent capacitance calculations are performed for all taps. The order of wires that form a particular tap is reorganized until the mutual capacitances for all taps are in practice equal. This is an iterative process and it is tracked using matrix algebra. The outcome is a suggested set of values for external capacitors that have to be applied between the taps in order to optimize the internal admittance loads.

Conclusion

An attempt was made to demonstrate the art of making a very accurate inductive voltage divider. The challenge was to present the material to undergraduate students. First, electromagnetic induction was introduced and magnetic field and its flux were presented so that the fundamental principles on which transformers work was explained. The next step was the ideal transformer definition. The following section that presented a real transformer was organized in such a way that each performance point of the ideal transformer was matched with practical realization. Techniques used to achieve the performance closest to the ideal case were explained in much detail. The state-of-the-art methods of two-stage transformer design and reducing the load of inter-winding capacitances were introduced. In order to more clearly present the material, actual two-stage transformer realization is shown in three steps: first the magnetizing winding is wound on a toroidal core and then the second core is placed on top. Both cores are put in a mu-metal toroidal box that provides magnetic shielding. Then the ratio windings are wound. Figures 3 and 4 give details of this construction. Also the process of reducing the admittance load of the inter-winding capacitances is demonstrated on a three-strand divider. All of the steps are presented in detail.

References

[1]   B. P. Kibble and G. H. Rayner, "Coaxial AC Bridges", Adam Hilger Ltd, TechnoHouse, Bristol, UK, 1984.

[2]   R. D. Cutkosky, "Techniques for Comparing Four Terminal Pair Admittance Standards", Journal of Research of the National Bureau of Standards – C. Engineering and Instrumentation, Vol. 74C, Nos. 3 and 4, pages 63-78, July-December 1970.

[3]   Z. Lu, M. Klonz and R. Bergeest, "Optimization of the Internal Admittance Load of and Inductive Voltage Divider for Low ratio Error", IEEE Transactions on Instrumentation and Measurement, Vol. 39, No. 3, pages 463-466, June 1990.

Biographical information

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